Economics 113 - Fall 2008 Professor Spearot
Introduction to Econometrics
Name John P. Answer Key

Midterm Exam # 1

The exam is closed book and closed notes. Please show your work step by step. Simple calculators may be used (no graphing or financial calculators and no cell phones)

You must show your work to receive full credit

Problem 1 (10 points)

You own three stocks. Their current weekly returns are -5, -4, and 3 percent. Further, their trading volumes are 25, 20 and 15 shares, respectively.

a. Calculate the mean return and mean volume. (2 points)

$$\hat{\mu}_R = \frac{1}{3}((-5)+(-4)+3) = \boxed{-2}$$

$$\hat{\mu}_V = \frac{1}{3}(25+20+15) = \boxed{20}$$

b. What is the correlation between return and volume for the three stocks? (3 points)

$$\hat{\sigma}_R = \sqrt{\frac{1}{2}\left((-5-(-2))^2+(-4-(-2))^2+(3-(-2))^2\right)}$$

$$= \sqrt{\frac{1}{2}(9+4+25)} = \sqrt{\frac{1}{2}(38)} = \boxed{\sqrt{19}}$$

$$\hat{\sigma}_V = \sqrt{\frac{1}{2}\left((25-20)^2+(20-20)^2+(15-20)^2\right)} = \sqrt{\frac{1}{2}(25+25)} = 5$$

$$\sigma_{R,V} = \frac{1}{2}\left((-5-(-2))(25-20)+(-4-(-2))(20-20)+(3-(-2))(15-20)\right)$$

$$= \frac{1}{2}\left((-3\cdot 5)+0+5\cdot(-5)\right) = \frac{1}{2}(-15-25) = -20$$

$$\rho_{RV} = \frac{(-20)}{(5)(\sqrt{19})} = \frac{\sigma_{RV}}{\sigma_R\sigma_V} = -\frac{20}{\sqrt{19}\cdot 5} = \boxed{-0.918}$$

c. Suppose that you add another stock, and the new mean return (of all 4 stocks) is -0.3. What is the return of the new stock? (2 points)

$$\hat{\mu}_R = \frac{1}{4}\left((-5)+(-1)+3+x\right) = -0.3$$

$$\underbrace{(-5)+(-1)+3}_{-6} + x = -1.2$$

$$x = -1.2 + 6$$

$$\boxed{x = 4.8}$$

d. Suppose that have a random variable X. Please **derive** the mean and standard deviation of a new variable Z, where $Z = \frac{X - \hat{\mu}_x}{\hat{\sigma}_x}$. (3 points)

$$\mu_z = \sum_{i=1}^{n} \frac{x_i - \hat{\mu}_x}{\hat{\sigma}_x} = \frac{1}{\hat{\sigma}_x}\sum_{i=1}^{n}(x_i - \hat{\mu}_x) = \frac{1}{\hat{\sigma}_x}\left(\sum_{i=1}^{n} x_i - \sum_{i=1}^{n}\hat{\mu}_x\right)$$

$$= \frac{1}{\hat{\sigma}_x}\left(\sum_{i=1}^{n} x_i - n\cdot\hat{\mu}_x\right)$$

$$= \frac{n}{\hat{\sigma}_x}\underbrace{\left(\frac{1}{n}\sum_{i=1}^{n} x_i - \hat{\mu}_x\right)}_{=0} = \boxed{0}$$

$$\hat{\sigma}_z^2 = \frac{1}{n-1}\sum_{i=1}^{n}(z_i - \hat{\mu}_z)^2$$

$$= \frac{1}{n-1}\sum_{i=1}^{n}(z_i)^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(\frac{x_i - \hat{\mu}_x}{\hat{\sigma}_x}\right)^2$$

$$= \underbrace{\left(\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \hat{\mu}_x)^2\right)}_{\hat{\sigma}_x^2}\frac{1}{\hat{\sigma}_x^2}$$

$$= \hat{\sigma}_x^2 \frac{1}{\hat{\sigma}_x^2} = 1$$

$$\hat{\sigma}_x = \sqrt{\sigma_x^2}$$

$$= \sqrt{1}$$

$$\boxed{\hat{\sigma}_x = 1}$$

Problem 2 (10 Points)
You roll a nine-sided die and three-sided die at the same time. After doing so, you record the sum of the numbers showing on the dice. Both dice are fair.

a. Please diagram this experiment (2 points)

	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12

b. What is the probability that the sum between the two dice is greater than 6? (3 points)

27 outcomes

15 Sum > 6

$$Pr(X>6) = \frac{15}{27} = \boxed{\frac{5}{9}}$$

c. What is the probability that the sum between the two dice is greater than 6 OR the nine-sided die is showing 5? (5 points)

$$Pr(X>6 \text{ ; } 9 \text{ showing } 5) = \frac{2}{27}$$

$$Pr(9 \text{ showing } 5) = \frac{1}{9}$$

$$Pr(X>6) = \frac{5}{9}$$

$$Pr(X>6 \cup 9 \text{ showing } 5) = Pr(X>6) + P(9 \text{ showing } 5) - Pr(X>6 \text{ ; } 9 \text{ showing } 5)$$

$$= \frac{5}{9} + \frac{1}{9} - \frac{2}{27} = \boxed{\frac{16}{27}}$$

Problem 3 (10 Points)
Professor Spearot's palette can be described as simple (at best). When tasting wine, he is able to detect three types of wine: *Dry*, *Fruity*, and *Bad*. During a recently tasting, he sampled wines from Santa Cruz, Napa, and Sonoma. For each region, the probabilities of receiving *Dry*, *Fruity*, and *Bad* wines were the following:

	Santa Cruz	**Napa**	**Sonoma**
Dry	0.3	0.5	0.3
Fruity	0.4	0.4	0.5
Bad	0.3	0.1	0.2

Regions are selected at random, where the probability of selecting a wine from Santa Cruz is 0.4, and the probability of selecting a wine from Sonoma is 0.35.

Given that Professor Spearot classifies a wine as *Bad*, what is the probability that the wine is from Napa?

$$Pr(Napa \mid Bad) = \frac{Pr(Napa \,\&\, Bad)}{Pr(Bad)}$$

$$Pr(Napa) = 1 - Pr(SC) - Pr(Sonoma) = 1 - 0.4 - 0.35$$

Using $Pr(Bad \mid Napa) = \dfrac{Pr(Bad \,\&\, Napa)}{Pr(Napa)}$

$$\Rightarrow Pr(Bad \,\&\, Napa) = Pr(Bad \mid Napa) \cdot Pr(Napa) = (0.1)(0.25) = \underline{0.025}$$

$$Pr(Bad) = Pr(Bad \,\&\, Napa) + Pr(Bad \,\&\, Santa Cruz) + Pr(Bad \,\&\, Sonoma)$$

$$= 0.025 + (0.3)(0.4) + (0.2)(0.35)$$

$$Pr(Bad) = \underline{0.215}$$

$$Pr(Napa \mid Bad) = \frac{0.025}{0.215} \Rightarrow \boxed{0.116}$$

Problem 4 (8 Points)
The average number of hours worked per day for UCSC students is characterized by a normal distribution with mean 3 and standard deviation 1.5.

a. What is the probability that a randomly selected student studies 4 hour per day? (2 points)

$$0$$

b. What is the probability that a randomly selected student studies more than 4 hours per day? (4 points)

$$Pr(Study > 4) = Pr(Z > z_4) = Pr\left(Z > \frac{4-3}{1.5}\right)$$

$$= Pr(Z > 0.667)$$

$$1 - Pr(Z < 0.67) = 1 - 0.7486$$

$$= 0.2514$$

c. What is the probability that a randomly selected student studies between 1 and 2 hours per day? (4 points)

$$Pr(Study \in (1,2)) = Pr(z_1 < Z < z_2)$$

$$= Pr\left(\frac{1-3}{1.5} < Z < \frac{2-3}{1.5}\right)$$

$$= Pr\left(-\frac{2}{1.5} < Z < -\frac{2}{3}\right) = Pr\left(-\frac{4}{3} < Z < -\frac{2}{3}\right)$$

$$= Pr\left(Z < -\frac{2}{3}\right) - Pr\left(Z < -\frac{4}{3}\right)$$

$$= \left(1 - Pr\left(Z < \frac{2}{3}\right)\right) - \left(1 - Pr\left(Z < \frac{4}{3}\right)\right)$$

$$= 0.2514 - (1 - 0.9082) = \boxed{0.1586}$$

Extra Credit: How many times do I need to flip a fair coin so that the probability of getting ALL heads is less than 0.0001? (5 Points)

$Pr(N \text{ Heads}) = (Pr \text{ Head})^N$

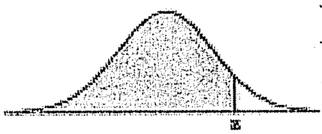
$0.0001 = (0.5)^N$

$\ln(0.0001) = N \ln(0.5) \Rightarrow N = 13.28$

Round ↑ $\boxed{N = 14}$

Helpful Formulas

$$\hat{\sigma}_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \hat{\mu}_x)^2 \qquad \hat{\sigma}_{xy} = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \hat{\mu}_x)(y_i - \hat{\mu}_y) \qquad \hat{\rho}_{xy} = \frac{\hat{\sigma}_{xy}}{\hat{\sigma}_x \hat{\sigma}_y}$$



Normal Distribution from -oo to Z

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990